

# A THEORETICAL AND EXPERIMENTAL COMPARISON OF DIRECTLY AND EXTERNALLY MODULATED FIBER-OPTIC LINKS

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## ABSTRACT

Analytic models of directly and externally modulated fiber-optic links have been derived and experimentally confirmed. The models have been employed to optimize the operating parameters of fiber-optic links. Experimental measurements on these optimized links indicated net incremental link power gains of +3 dB for direct modulation and +11 dB for external modulation. The implications of these optimizations on other measures of link performance, such as bandwidth and noise figure, are also presented.

## I. INTRODUCTION

Fiber-optic links are beginning to be used in an increasing number of RF system applications. A prime motivation for using fiber-optic links is that optical fibers per se offer a number of significant advantages, such as their low loss and high bandwidth, over other methods of conveying RF signals. Unfortunately, the many advantages of optical fibers are presently masked in fiber-optic links by limitations of the RF-to-optical and optical-to-RF conversion processes. For example, although the loss of the fiber may be less than 1 dB/km, the RF-optical-RF conversion processes typically result in a zero-length link loss (i.e.,  $|S_{21}|$ ) of 20 to 40 dB. In order to understand the basis for these losses, incremental models for the relevant electro-optic devices - laser, external modulator and photodetector - will be developed. From examination of these models, it will become apparent which techniques would be most effective at reducing this loss and what the tradeoffs will be with other link parameters, such as bandwidth and noise figure.

## II. INCREMENTAL MODELS

### A. Semiconductor Laser<sup>(1)</sup>

In general the optical output power  $\bar{P}_0$  from a semiconductor laser can be related to the current  $I_L$  flowing through the laser by the slope efficiency:

$$\eta_L(I_L) = d\bar{P}_0/dI_L \quad (1)$$

For linear modulation, a bias current  $I_B$  must be provided that is greater than the laser threshold current  $I_T$ . The modulation current  $I_{LM}$  is superimposed on  $I_B$ , resulting in a total laser current  $I_L = I_B + I_{LM}$ . If  $I_{LM} \ll I_B$  then  $\eta_L(I_L) \approx \eta_L(I_B) = \eta_{LB}$ , i.e., a slope efficiency that is independent of  $I_{LM}$ . Under these conditions, the incremental model for a semiconductor laser is simply

$$P_0 = \eta_{LB} I_{LM} \quad (2)$$

### B. Balanced Mach-Zehnder (MZ) External Modulator<sup>(2)</sup>

In general the optical output  $\bar{P}_0$  from this type of modulator is related to the optical input power  $\bar{P}_i$  and the

voltage  $V_E$  applied to modulator electrodes in the following manner:

$$P_0 = \frac{P_i}{2} \left( 1 + \cos \frac{\pi V_E}{V_\pi} \right) \quad (3)$$

where  $V_\pi$  is a parameter of the modulator that depends upon the specifics of its design. As with the laser, in order to accomplish linear modulation, a bias voltage  $V_{EB}$  must be supplied to the modulator with the desired modulation voltage  $V_M$  superimposed on top of the bias:  $V_E = V_{EB} + V_M$ . Since the transfer function for these devices is periodic, any of a number of bias voltages could be chosen in principle; typically the lowest positive one is used, i.e.,  $V_{EB} = V_\pi/2$ . For  $V_M \ll V_{EB}$ , the incremental transfer function for a MZ external modulator is

$$P_0 = \frac{P_i}{2} \left( -\frac{\pi V_M}{V_\pi} \right) \quad (4)$$

### C. PIN Photodetector<sup>(3)</sup>

Under reverse bias, the current  $I_D$  that flows through a PIN photodiode is related to the optical power  $\bar{P}_{0D}$  incident on the photodetector via the slope efficiency:

$$\eta_D = dI_D/d\bar{P}_{0D} \quad (5)$$

As this relationship typically is linear over approximately seven orders of magnitude,  $\eta_D$  is independent of  $\bar{P}_{0D}$  over this range. Consequently, the above relationship also holds for incremental signals.

## III. LINK TRANSFER FUNCTIONS AND BANDWIDTH

A convenient measure of link RF-to-RF performance is the link transducer gain,  $G = P_{OUT}/P_{IN,A}$  where  $P_{OUT}$  is the RF power delivered to the load at the detector end of the link and  $P_{IN,A}$  is the available RF power from the RF source at the input end of the link. Because of the optical isolation between the optical source and detector ends of the link,  $G$  can be expressed as the product of three separately determinable components:

$$G = T_S T_{OD}^2 T_D \quad (6)$$

$T_S$  is the incremental optical source (laser or external modulator) efficiency; it will be expressed as  $P_0^2/P_{IN,A}$  where  $P_0$  is the modulated optical power from the optical source.  $T_D$  is the incremental detector efficiency; it will be expressed as  $P_{OUT}/P_{0D}^2$  where  $P_{0D}$  is the modulated optical power incident on the detector. In both these cases the square of optical power appears because for all the electro-optic devices under consideration here, optical power is proportional to either an RF voltage or current.  $T_{OD}$  is the link optical efficiency; it will be expressed as  $P_{0D}/P_0$ .  $T_{OD}$  includes all

factors affecting the transmission of light from the source to the detector such as source-to-fiber coupling efficiency, fiber and connector attenuation and fiber-to-detector coupling efficiency.

It turns out that the intrinsic impedance of the electro-optic devices discussed above are not well matched to the 50  $\Omega$  impedance used in many RF systems. Consequently, in developing these link models, it is desirable to include some provision for impedance matching. An additional benefit of matching, if done appropriately, is the reduction or even the elimination of the link RF-to-RF insertion loss. It has been found that a convenient mechanism for investigating the potentials and limitations of impedance matching is the ideal transformer.

#### A. Semiconductor Laser

The incremental circuit model to be analyzed here is shown in Figure 1. This model assumes that the laser is

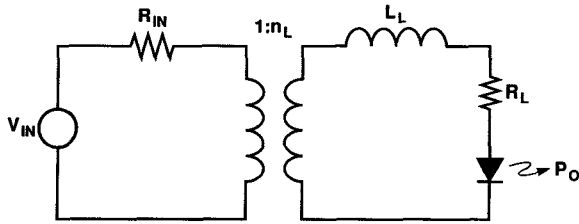


Figure 1. Circuit for analyzing the incremental modulation efficiency and bandwidth of a semiconductor laser.

operated at frequencies well below its relaxation resonance frequency.<sup>(4)</sup> Consequently the dominant circuit elements are the resistance in series with the diode junction  $R_L$  and the bond wire inductance  $L_L$ . Further it can be shown that for  $I_B > I_T$  the incremental voltage drop across the laser diode junction is negligible in comparison with the incremental voltage drop across  $R_L$ .<sup>(5)</sup> With these assumptions in mind, a straightforward circuit analysis yields the following relationship between  $P_O$  and  $V_{IN}$ :

$$P_O = \frac{\eta_{LB} n_L}{R_{TI} (s n_L^2 L_L / R_{TI} + 1)} V_{IN} \quad (7)$$

where  $R_{TI} = n_L^2 R_L + R_{IN}$ ,  $n_L$  is the turns ratio of the transformer and  $s = \alpha + j\omega$ , is the complex frequency. The RF power available from the source is

$$P_{IN,A} = V_{IN}^2 / 4R_{IN} \quad (8)$$

Dividing the square of equation (7) by equation (8) yields the incremental modulation efficiency for a semiconductor laser:

$$\frac{P_O^2}{P_{IN,A}} = \frac{4R_{IN} \eta_{LB}^2 n_L^2}{R_{TI}^2 (s n_L^2 L_L / R_{TI} + 1)^2} \quad (9)$$

An important special case is when  $R_L$  is matched to  $R_{IN}$  via  $n_L$ , i.e., when  $n_L^2 R_L = R_{IN}$ . Under this condition equation (9) reduces to

$$\left( \frac{P_O}{P_{IN,A}} \right)_M = \frac{\eta_{LB}^2}{R_L (s L_L / 2R_L + 1)^2} \quad (10)$$

It is evident from equation (10) that to maximize this transfer function, and thereby reduce link RF-to-RF insertion loss, one wants to maximize  $\eta_{LB}$  and to minimize  $R_L$  - to the extent possible while still achieving a match to  $R_{IN}$  via  $n_L$ . However, a reduction in  $R_L$  will lead to decreased bandwidth.

#### B. MZ External Modulator

The incremental circuit model incorporating a MZ modulator is shown in Figure 2. The series elements  $R_{SM}$  and  $C_M$  represent the resistance and capacitance, respectively, of the modulator electrodes. The parallel resistor  $R_{PM}$  represents the resistive termination that is commonly used with these modulators. A straightforward circuit analysis yields the following relationship between  $P_O$  and  $V_{IN}$ :

$$P_O = \left( \frac{P_I \pi}{2V_{IN}} \right) \left( \frac{n_M R_{PM}}{R_{T2}} \right) \left( \frac{1}{s C_M R_{T3} + 1} \right) V_{IN} \quad (11)$$

where

$$R_{T2} = n_M^2 R_{PM} + R_{IN}$$

and

$$R_{T3} = \frac{R_{IN} (R_{PM} + R_{SM}) + n_M^2 R_{PM} R_{SM}}{R_{T2}}$$

In general RF matching would require

$$R_{IN} = n_M^2 \text{Re} [R_{PM} \parallel (R_{SM} + 1/sC_M)]$$

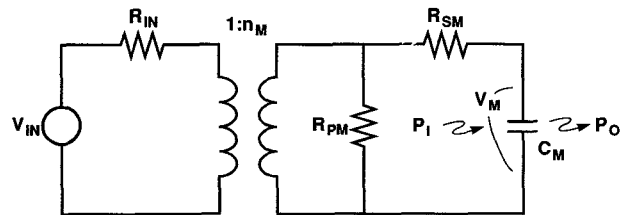


Figure 2. Circuit for analyzing the incremental modulation efficiency and bandwidth of a MZ external modulator.

Significant insight into the matching options can be gained by considering two limiting cases:

- 1.) parallel match, where  $n_M^2 R_{PM} = R_{IN}$
- 2.) series match, where  $n_M^2 R_{SM} = R_{IN}$ .

**1. MZ with parallel match** To achieve a parallel match it is required that  $|R_{SM} + 1/sC_M| \gg R_{PM}$ . However, since in general  $R_{SM} < R_{PM}$ , the condition for parallel match can be refined to  $|sC_M|^{-1} \gg R_{PM}$ . Using this

condition to simplify equation (11), then squaring the result and dividing by equation (8) yields the incremental modulation efficiency for a parallel-matched MZ modulator:

$$\left( \frac{P_0}{P_{IN,A}} \right)_{M,P} = \left( \frac{P_I \pi}{2V_\pi} \right)^2 \left( \frac{R_{IN}}{n_M^2} \right) = \left( \frac{P_I \pi}{2V_\pi} \right)^2 R_{PM} \quad (12)$$

To maximize this modulation efficiency, one must maximize  $P_I$ , minimize  $V_\pi$ , and make  $R_{PM}$  as large as possible consistent with the parallel match and frequency response requirements. Also note that this efficiency is maximized for  $n_M < 1$ ; consequently one actually wants a step-up transformer, rather than the step-down one indicated in Figure 2. Traditionally, the parallel match case has been used with  $n_M = 1$ , i.e.,  $R_{PM} = R_{IN}$ .

**2. MZ with series match** To achieve a series match, it is required that  $|R_{SM} + 1/sC_M| < R_{PM}$ . Since  $R_{SM} < R_{PM}$ , a tighter constraint is  $|sC_M|^{-1} < R_{PM}$ , i.e., the series approximation is valid at least for  $\omega > (C_M R_{PM})^{-1}$ . In order to match to  $R_{SM}$ , the  $R_{SM} - C_M$  loop cannot be operated at frequencies where  $C_M$  appears to be an open circuit. Therefore the minimum frequency of operation for a series match is  $\omega > (C_M R_{SM})^{-1}$ . Thus while the parallel match must operate in a low-pass mode, the series match must operate in a high-pass mode. By using the above conditions to simplify equation (11), squaring the result and dividing by equation (8), we get the incremental modulation efficiency for a series matched MZ modulator:

$$\left( \frac{P_0}{P_{IN,A}} \right)_{M,S} = \left( \frac{P_I \pi}{2V_\pi} \right)^2 \left( \frac{n_M^2}{s^2 C_M^2 R_{IN}} \right) = \left( \frac{P_I \pi}{2V_\pi} \right)^2 \frac{1}{s^2 C_M^2 R_{SM}} \quad (13)$$

In contrast to the parallel match case, for the series match case the incremental efficiency is maximized by increasing the step-down ratio, i.e.,  $n_M > 1$  and minimizing  $R_{SM}$ , both to the extent permitted by the series match and frequency response requirements. Although equation (13) was derived for a specific circuit, it can be shown<sup>(6)</sup> that equation (13) represents the optimum modulation efficiency for any passive matching circuit.

### C. PIN Photodiode

The incremental circuit model for the photodiode is shown in Figure 3, where  $R_{SD}$  is the photodiode series resistance and  $C_D$  is the diode capacitance. A straightforward analysis of the components in Figure 3 yields the

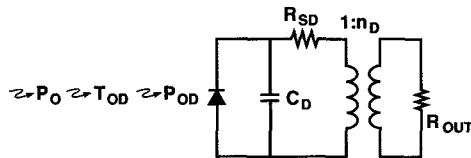


Figure 3. Circuit for analyzing the incremental modulation efficiency and bandwidth of a PIN photodiode.

photodiode incremental modulation efficiency:

$$\frac{P_{OUT}}{P_{OD}} = \frac{n_D^2 R_{OUT} \eta_D^2}{[sC_D(R_{SD} + n_D^2 R_{OUT}) + 1]^2} \quad (14)$$

As with the previous cases, equation (14) makes clear the tradeoff between maximizing modulation efficiency - by maximizing the turns ratio  $n_D$  and the simultaneous reduction in bandwidth that result from increases in  $n_D$ . The bandwidth consequence follows because  $R_{SD}$  is the same order of magnitude as  $R_{OUT}$ .

## IV. NOISE

In this section the noise factor of directly and externally modulated links will be derived. For all cases there are three dominant noise sources: the thermal noise of the resistive component of the source or modulator, the shot noise at the photodetector, and the laser's relative intensity noise (RIN). The thermal noise of the output load  $R_{OUT}$  is usually negligible compared with the shot noise; therefore it has been omitted.

The mean-square thermal-noise voltage  $E_T^2$  of a resistor  $R$  per unit bandwidth is  $4kTR$ . The mean-square shot-noise current  $I_{SN}^2$  per unit bandwidth can be expressed as  $I_{SN}^2 = 2q\eta_D T_{OD} \bar{P}_0$ , where  $\bar{P}_0$  is the average optical power of the source. The laser RIN depends upon a number of laser parameters and operating conditions. In general, for semiconductor lasers  $RIN(\bar{P}_0)$  decreases as  $\bar{P}_0^{-3}$  for  $I_B > I_T$  and as  $\bar{P}_0^{-1}$  for  $I_B \gg I_T$ . The RIN for solid-state lasers is negligible.

The link noise factor  $F$  can be expressed as

$$F = 1 + N_{NO}/N_I G \quad (15)$$

where  $N_{NO}$  is the noise power at the link output with the RF source noise power  $N_I$  set equal to zero and  $G$  is the link transducer power gain. Link models, including the dominant noise sources, are shown in Figure 4.

The results of an analysis for each of these circuits can be expressed in the following general form:

$$F = 1 + \frac{n^2 E_T^2}{E_T^2} + \frac{4R_{IN} R_{OUT} n_D^2}{G E_T^2} (I_{SN}^2 + I_{RIN}^2) \quad (16)$$

where  $n$  is the turns ratio for matching the RF source into the laser or modulator. An important special case occurs when the laser or modulator resistance  $R$  is matched to the RF source and the RF source and load are matched, i.e., when

$$n^2 R = R_{IN} = R_{OUT} \quad (17)$$

Applying condition (17) to equation (16) results in the matched noise factor

$$F_M = 2 + \frac{n_D^2 R_{IN}}{kTG_M} (I_{SN}^2 + I_{RIN}^2) \quad (18)$$

where  $G_M$  is  $G$  under the matched conditions of equation (17). The factor of 2 (which corresponds to a noise figure of 3 dB) represents the fundamental limit for a passively matched input. Clearly, reductions in the link loss (i.e., increases in  $G_M$ ) are necessary in order for  $F_M$  to approach the 3 dB limit.

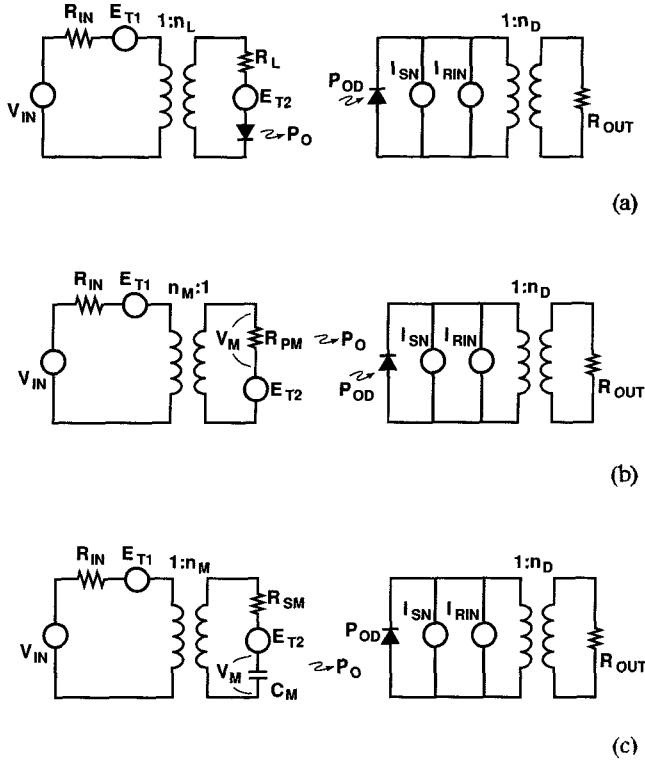


Figure 4. Circuits for analyzing the noise figure of a (a) directly modulated link and an externally modulated link with (b) parallel and (c) series match.

In order to calculate  $F_M$ , explicit expressions for  $G_M$  for each of the links discussed in Section III are required. For the directly modulated link the matched link transfer function  $G_{D,M}$  is a product of the laser and photodiode modulation efficiencies - equations (10) and (14), respectively. Over a frequency range where the frequency dependent terms in both these equations are negligible,  $G_{D,M}$  can be expressed as

$$G_{D,M} = n_L^2 \eta_L^2 T_{OD}^2 \eta_D^2 n_D^2. \quad (19)$$

For the externally modulated link, the gain expression will depend upon the matching option. Under parallel match conditions, the matched link transfer function  $G_{EP,M}$  is the product of equations (12) and (14). Again neglecting frequency dependent terms, we get

$$G_{EP,M} = R_{PM}^2 n_M^2 \left( \frac{P_I \pi}{2V} \right)^2 T_{OD}^2 \eta_D^2 n_D^2. \quad (20)$$

Under series match conditions, the matched link transfer function  $G_{ES,M}$  is the product of equations (13) and (14). Neglecting frequency dependent terms, we get

$$G_{ES,M} = \frac{n_M^2}{s^2 C_M^2} \left( \frac{P_I \pi}{2V} \right)^2 T_{OD}^2 \eta_D^2 n_D^2. \quad (21)$$

Although equations (19-21) have the same general form, there is a fundamental difference between equation (19) and (20),(21). In the latter two equations the gain depends upon the square of the average optical power since  $P_I / 2 \propto P_O$ , whereas equation (19) is independent of  $P_O$ . The importance of this observation can be appreciated by recalling from the introduction to Section IV that shot noise is proportional to  $P_O$ . Consequently, its effects can be reduced in principle to arbitrarily low levels by increasing  $P_O$ , since  $G_{E,M} \propto P_O^2$ .

## V. CONCLUSIONS

Incremental device models of the basic electro-optic components used in fiber-optic links - diode laser, external modulator and photodiode - were derived. Incremental link models were formed from these device models. For both direct and externally modulated links it can be seen that there are similar tradeoffs between link transducer gain  $G$  and RF bandwidth. Examination of these models also indicates that there is no fundamental reason why  $G$  must be less than one. For both types of links, impedance matching can be used to reduce link loss (thereby increasing  $G$ ). Externally modulated links offer an additional mechanism for increasing  $G$  since in these links  $G$  depends upon the square of the average optical power. Increasing  $G$  by any of the methods presented above in either type of link will improve the link noise figure. However, for the externally modulated link,  $G$  can be made to increase faster than increases in the photodetector shot noise, thereby permitting, in principle, link noise figures that are arbitrarily close to the fundamental limit.

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## REFERENCES

- (1) G. P. Agrawal and N. K. Dutta, *Long-wavelength Semiconductor Lasers*, Van Nostrand, NY, 1986, Chapter 2.
- (2) R. V. Schmidt, "Integrated Optics Switches and Modulators," in *Integrated Optics Physics and Applications*, S. Martellucci and A. N. Chester, eds., Plenum Press, NY, 1987.
- (3) J. M. Senior, *Optical Fiber Communications*, Prentice Hall, Englewood Cliffs, NJ, 1985, Chapter 8.
- (4) G. P. Agrawal and N. K. Dutta, op. cit., pp. 256-263.
- (5) Ibid., pp. 184-186.
- (6) G. E. Betts, L. M. Johnson and C. H. Cox, "High Sensitivity Bandpass RF Modulator in LiNbO<sub>3</sub>," *Proc. SPIE O-E Fiber/LASE '88*, Boston, MA, Sept. 1988.